Extraordinary Interactions between Light and Matter Determined by Anomalous Weak Values

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Some predictions regarding pre- and post-selected states are far-reaching, thereby requiring validation with standard quantum measurements in addition to the customary weak measurements used so far, as well as other advanced techniques. We go further pursuing this goal, proposing two thought experiments which incorporate novel yet feasible validation methods of unconventional light-matter interactions. An excited atom traverses a Mach-Zehnder interferometer (MZI) under a special combination of pre- and post-selection. In the first experiment, photons emitted by the superposed atom, after being hit by two laser beams, are individually counted. Despite the interaction having definitely taken place, as revealed by the atom becoming ground, the numbers of photons emitted from each arm of the MZI are predicted, at the ensemble level, to be different from those expected with standard stimulated emission. In the second experiment, the atom spontaneously emits a photon while still in the MZI. This photon later serves as a strong measurement of the atom's energy upon hitting a photographic plate. The experiment is repeated to enable an interference effect of the emitted photons. Interestingly, the latter gives the appearance that the photons have been emitted by the atom from a position much farther from the two MZI arms L and R, as if in a "phantom arm" R'. Nevertheless, their time of arrival is similar to that of photons coming from L and R. These experiments also emphasize the key role of anomalous weak values in determining light-matter interactions. In fact, they present a straightforward realization of an entity we term counter-particles, namely pre- and post-selected states acting as if they have negative physical variables such as mass and energy. The novel verification methods we suggest for testing these predictions resemble weak measurements in some aspects, yet result from definite atomic transitions verified by the detected photons.

The Two-State-Vector Formalism (TSVF) [1, 2] offers a simple yet very efficient and fruitful method of studying quantum phenomena. Classical physics enables prediction of a future state based on the system's initial conditions. Conversely, one can retrodict past states on the basis of final conditions. The two methods are equivalent, hence each is redundant to the other. Not so in quantum mechanics: Prediction alone, using the pre-selected wavefunction $|\Psi\rangle$, and retrodiction alone, with the post-selected wavefunction $|\Phi\rangle$, give only partial (and sometimes conflicting) information due to quantum uncertainty. However, their *combination* in the form of the two-state $\langle\Phi| \ |\Psi\rangle$ gives much more information [1, 2]. This information is available through inference of the weak value [3] of any operator $\hat{A}$,

$$\langle\hat{A}\rangle_w = \frac{\langle\Phi|A|\Psi\rangle}{\langle\Phi|\Psi\rangle}. \tag{1}$$

Moreover, when the two boundary conditions markedly differ from one another, their combined information gives rise to "anomalous weak values," (also called "superweak values" [4]) i.e., too large/small or even complex [5–9].

These intriguing values have been demonstrated, so far, mainly with the aid of weak measurements [3] (see however a recent claim of strongly measuring weak values [10] and the corresponding comment [11]). Although being somewhat controversial as an experimental tool [12–14], weak measurements have led to many practical (e.g. [15–19]) and conceptual (e.g. [20–28]) achievements. Close in spirit to this paper was the demonstration of spontaneous-emission-based weak measurements [29]. Yet, since all the above measurements involve weak coupling between the measuring pointer and measured system, thereby afflicted with inherent quantum noise, they have sometimes been explained away, although erroneously (see for instance [30–32]), as artifacts of noise [33]. The introduction of projective ("strong") quantum measurements for the validation of TSVF predictions [34–37] was therefore a major advance, immune to above objections and offering a deeper understanding of the quantum realm.

Following are two experiments of this kind, where curious predictions of TSVF are verified with the aid of both weak and strong measurements. These are performed after the measured atom has definitely changed its state (as indicated by the post-selection). Then the photons that have interacted with the atom are detected, revealing what has occurred between the pre- and post-selection. The overall effect, like many fundamental quantum mechanical ones, is proved on the basis of a sufficiently large ensemble. To the best of our knowledge, not only the effects, but also the validation techniques are novel, yet highly feasible. We shall nevertheless focus on concepts, rather than technical details.

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This paper is organized as follows. We first present the basic setup and then analyze two different thought experiments, one involving stimulated emission of radiation and the other incorporating spontaneous emission. We then conclude with some general consequences.

A. Pre- and Post–Selected States of Atoms

An excited atom traverses an atomic Mach-Zehnder interferometer (MZI) (Fig. 1) such that its initial spatial superposition state, determined by the MZI's first beam-splitter (BS), is uneven

$$|\Psi\rangle = \frac{4}{5}|R\rangle - \frac{3}{5}|L\rangle. \tag{2}$$

Upon exiting the MZI, the atom is post-selected for a special case with respect to this pre-selection. For example, in contrast to the first $16/25$–$9/25$ atomic BS within the MZI, let the second BS be $1/2$–$1/2$. Then post-select the atom for detection events by the left-hand detector D rather than the normally expected C.

Next compute the atom's backwards evolution. It is unitary, hence the post-selection carries information about the particle's preceding evolution just as the pre-selection (preparation) gives the evolution that follows. The retrodiction about the particle's past state within the MZI is therefore

$$|\Phi\rangle = \frac{1}{\sqrt{2}}|R\rangle + \frac{1}{\sqrt{2}}|L\rangle. \tag{3}$$

According to the TSVF, the pre- and post-selected states, though apparently incompatible, are equally valid. Moreover, when (2) and (3) are inserted into (1), together they give rise to anomalous weak values that have prevailed during the intermediate time interval within the MZI,

$$\begin{aligned}\langle\hat{\Pi}_R\rangle_w &= \frac{\left(\left(\langle R| + \langle L|\right)\hat{\Pi}_R\left(4|R\rangle - 3|L\rangle\right)/\sqrt{50}\right.}{\left(\left(\langle R| + \langle L|\right)\left(4|R\rangle - 3|L\rangle\right)/\sqrt{50}\right.} = 4 \\ \langle\hat{\Pi}_L\rangle_w &= \frac{\left(\left(\langle R| + \langle L|\right)\hat{\Pi}_L\left(4|R\rangle - 3|L\rangle\right)/\sqrt{50}\right.}{\left(\left(\langle R| + \langle L|\right)\left(4|R\rangle - 3|L\rangle\right)/\sqrt{50}\right.} = -3,\end{aligned} \tag{4}$$

where $\hat{\Pi}_{R/L}$ is a projector on the MZI's right/left arm. Effectively, this is a description of "four atoms" on one arm and "minus three atoms" on the other. Following are two novel measurement techniques that reveal these weak values.

B. Stimulated Emission indicating the Atom's Odd Weak Values

For validating the above extraordinary weak values, the atom's excited state comes to our aid. Between preparation and post-selection, let the atom be hit by two laser beams (having zero mutual overlap) directed towards its two possible locations within the MZI (Fig. 1). Within the Gedankenexperiment's scope, let the interaction between the photons and the atom be such that it leads to stimulated emission of radiation with probability approaching 1. Atoms that have nevertheless remained excited are selected out too. We are therefore assured that among the photons eventually detected by detectors L and R there is one additional photon emitted by the excited atom via stimulated emission.

The laser beams are described by the following coherent state, represented via the real quadrature q (in optical phase space):

$$\psi(q) = (2\pi)^{-1/2} \exp\left[-\frac{(q-q_0)^2}{4}\right], \tag{5}$$

where $q_0 \gg 1$, which is typically the case in the lab.

The interaction with the excited atom is assumed to change this state into:

$$\tilde{\psi}(q) = (2\pi)^{-1/2} \exp\left[-\frac{(q-q_0')^2}{4}\right], \tag{6}$$

where $q_0' = q_0 + 1/2q_0$ accounts for the additional photon (in suitable units the change in the number of photons is $q_0^2 - q_0'^2$ which equals to 1 up to a negligible factor of $1/4q_0^2$).

In the sub-ensemble of atoms under successful post-selection, the state of emitted photons for the right arm is [38]

$$\psi^R_{\text{em}}(q) = \langle\hat{\Pi}_R\rangle_w (2\pi)^{-1/2} \exp\left[-\frac{(q-q_0')^2}{4}\right] + \left(1 - \langle\hat{\Pi}_R\rangle_w\right)(2\pi)^{-1/2} \exp\left[-\frac{(q-q_0)^2}{4}\right], \tag{7}$$

where $\hat{\Pi}_R$ is a projector on the MZI's right arm. We have used the weak value because the state of the atoms is pre- and post-selected. It is important to note that when $q_0 \gg 1$, the stimulated emission (namely the 1 photon added by the atom to the initial average number of photons emitted by the source) cannot provide which-path information. Therefore, although this interaction between light and matter is "strong", the atoms' spatial superposition barely changes due to the uncertainty in the number of photons. It is this aspect which makes the proposed technique resemble weak measurement, yet with the advantage that the atom's energy was strongly measured and thus allowed observation of an anomalous stimulated emission effect.

Repeat this procedure sufficiently many times to make a reliable statistical estimate of the number of photons emitted from the atom's two possible locations on the MZI's two paths.

Substituting $\langle\hat{\Pi}_R\rangle_w = 4$ in (7) and using the first order approximation of the exponential function with $q_0 \gg 1$, results in

$$\psi^R_{\text{em}}(q) \approx (2\pi)^{-1/2} \left(1 - \frac{q-q_0}{q_0}\right) \exp\left[-\frac{(q-q_0)^2}{4}\right] \approx (2\pi)^{-1/2} \exp\left[-\frac{(q-q_0'')^2}{4}\right], \tag{8}$$

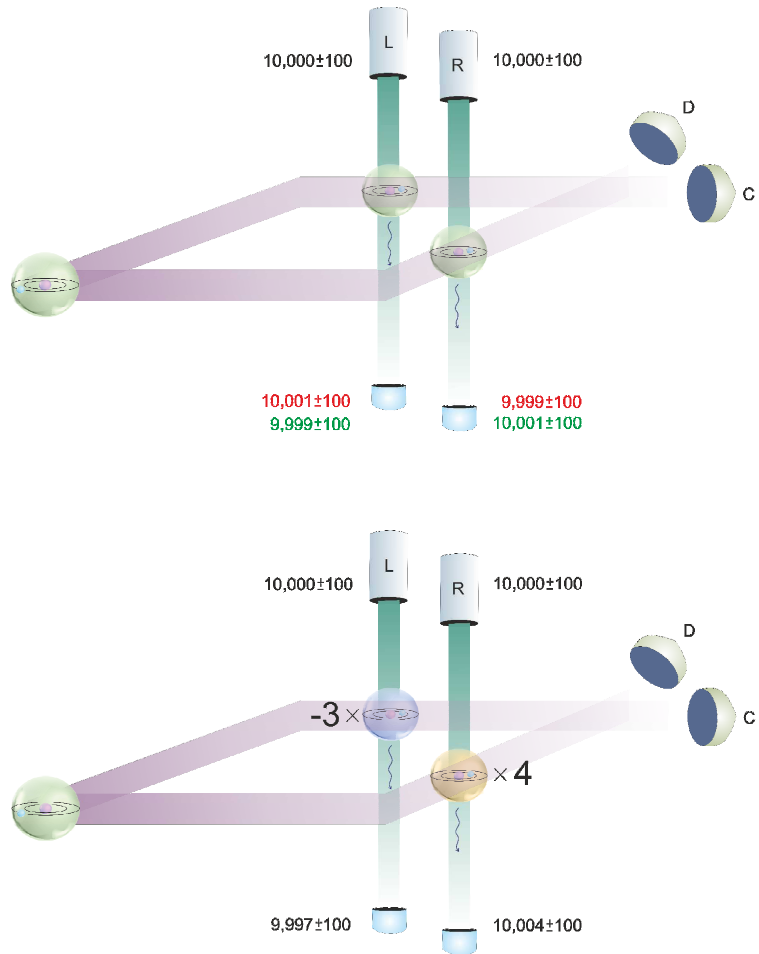

FIG. 1: An atom (whose state is pre- and post-selected) traverses an MZI and interacts with two photonic beams having a large uncertainty in their photon number. The excited atom undergoes stimulated emission, becoming ground. The number of photons detected at either L and R does not give strong "which-path" information regarding the atom, hence the atomic interference is hardly impaired. Some average numbers have been inserted for illustrating the effect. Upper panel: Ordinary post-selection (both BSs with 50% transmission coefficient). The extra photon is detected by either L and R, making their outcomes correlated yet much below the level of noise. Lower panel: Special pre- and post-selection (BS1 with 64% and BS2 with 50% transmission coefficients). This time the weak values of the particle numbers on each arm are -3 and 4. The uncertainty in the number of photons is still greater, but these weak values can be statistically inferred from the average number of hits in each detector using a large enough ensemble.

where $q_0'' = q_0 + 2/q_0$, i.e. $q_0''^2 - q_0^2 \approx 4$, meaning that in the right-arm position, the atom has reacted to the laser beam as if it *were 4 atoms* (the new conditional mean has been approximately shifted by 4), emitting 1 photon each. Even more striking is the similarly derived

$$\psi_{\text{em}}^{L}(q) \approx (2\pi)^{-1/2} \exp\left[-\frac{(q - q_0''')^2}{4}\right], \tag{9}$$

where $q_0''' = q_0 - 3/2q_0$, implying that *the effective -3 left-arm-atoms have absorbed one photon each,* *leading on average to a decrease in the number of photons on this side.*

As a consistency check, we note that the anomalous numbers of photons 4 and -3 add up to 1 as they should. Some numbers have been given in Fig. 1 for illustrating the effect.

To summarize, under a special combination of pre- and post-selections, the atom traversing the MZI is retrodicted to possess odd properties. Consider first the familiar superposition $|\Psi\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$, that means: upon performing a projective which-path measurement, there is either 1 atom in the right arm or 1 in the left. Hence, had such an atom absorbed the laser beam during its passage through the MZI, it would emit one photon from either side. In our subensemble, however, the atom presents a curious effective interaction: *There are either* 4 *atoms in the right arm or* -3 *(minus three) atoms in the left, as indicated by the number of detected photons.*

Generalizing, for the pre-selected state

$$|\Psi\rangle = \left(\alpha|R\rangle - \beta|L\rangle\right)/\sqrt{\alpha^2 + \beta^2},$$

where $\alpha = \beta + 1 \in \mathbb{R}$, followed by the post-selected state of (3), we would find on average α extra photons in the right detector and a deficit of β photons on the left detector (as long as $q_0 \gg \alpha$). Similarly, if the superposed atom is ground and we try to excite it, then we expect to find excessive photons on the negative arm.

C. The Atom's Spontaneous Emission of Radiation Appearing to Originate from a Phantom Position

Consider again our excited atom's pre- and post-selected state in Eqs. 2,3 (Fig. 2). The above predicted appearance of an effective negative property can be further studied within a complementary scenario, now with the aid of an interference effect that reveals another unexpected phenomenon. This time, make the time spent by the atom within the interferometer much larger than its half-life time, such that between the pre- and post-selection it is most likely to undergo spontaneous rather than stimulated emission either at $x = -d$ or $x = d$ (which correspond to the left/right arms of the interferometer, respectively). Make the emitted photon's wavelength λ much larger than the distance between the interferometer arms $2d$. Consequently, again, radiation cannot reveal which path the atom took.

Photons emitted from the right/left-arm-atom are described by the following spatial distributions: $g^{R/L}(x) = \phi(x \pm d)$, having the same functional form, yet centered around $\pm d$, respectively, with a typical waist of λ (for simplicity, $\phi(x)$ can be thought of as being a Gaussian with a standard deviation of order λ). The photon's long wavelength allows interference between these two distributions without disclosing which path information. Let us take advantage of this effect by detecting these photons with the aid of a

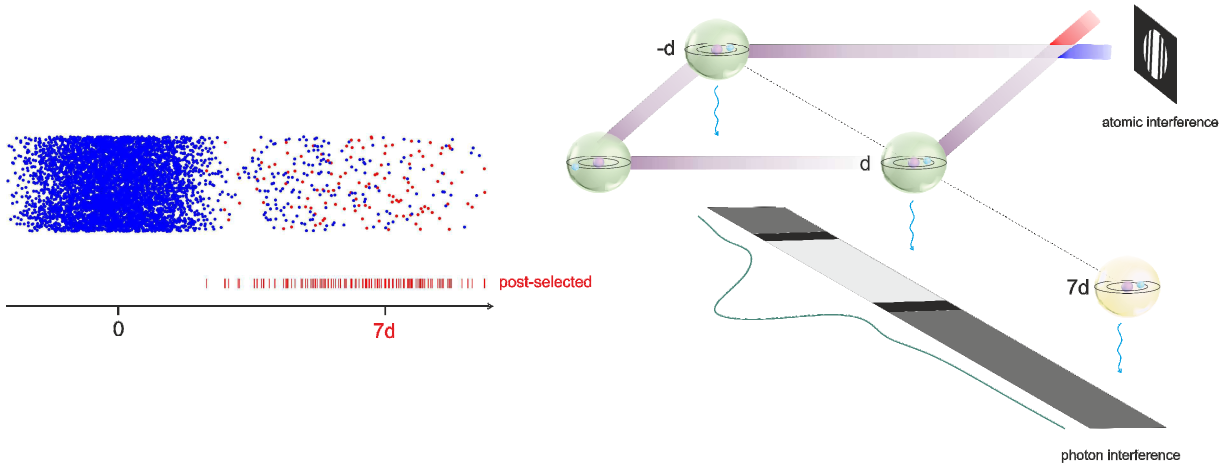

FIG. 2: An excited atom traversing an atomic MZI, exhibiting interference by ending at the blue detector. The atom is delayed within the MZI to allow it to emit a photon later absorbed by a photographic plate. The photon's long wavelength (in comparison to the size of the MZI) does not allow the presence of "which path" information regarding the atom. The experiment is repeated sufficiently many times for statistical averaging. The two arms of the MZI are so close to each other compared to the photon's wavelength such that the overall effect is like that of a wide single slit. Consider the same excited atom under the above special pre- and post-selection, namely, with 64% and 50% transmission coefficients at the first and second beam splitters followed by detection at the red detector. This time, the photons detected on the screen, upon statistical averaging, correspond to a spontaneous emission from an atom effectively located far outside the interferometer. This, however, is an interference phenomenon of anomalous weak values.

photographic plate, just as in the double-slit experiment. We therefore have two interference effects, one manifested by the atom itself exiting from the MZI upon post-selection and the other by the photons it has emitted earlier.

To better comprehend the predicted effect, let us drop for a while the special post-selection and consider an excited atom traversing an ordinary MZI while emitting a photon. Here, post-selection is trivial, namely, exactly reuniting the two atomic wave-function halves split by the first BS (e.g., both beam splitters being 50% transparent) and detecting them in the constructive interference arm C. We expect a single-slit-like pattern, indicating in fact two slits very close to one another (Fig. 2):

$$\phi_{\text{Tot}}(x) = \frac{1}{\sqrt{2}}\left[\phi(x+d) + \phi(x-d)\right]. \tag{10}$$

Now bring back the above post-selection (3). Statistically averaging again, the total distribution of photons for the pre- and post-selected cases would be

$$g(x) = \langle \hat{\Pi}_L \rangle_w \phi(x+d) + \langle \hat{\Pi}_R \rangle_w \phi(x-d) = -3\phi(x+d) + 4\phi(x-d) = [-3\exp(2i\hat{p}d/\hbar) + 4]\phi(x-d)$$
$$\approx (1 - 6i\hat{p}d/\hbar)\phi(x-d) \approx \exp(-6i\hat{p}d/\hbar)\phi(x-d) \approx \phi(x-7d), \quad (11)$$

where $\hat{p}$ is the photon's momentum and $\phi(x \pm d)$ is the wave emitted from L/R, respectively. This means that these particular photons exhibit an interference pattern suggesting that each was emitted by an atom residing in the "phantom position" located at $R' = 7d$, i.e. far outside the MZI (see Fig. 2). This is another manifestation of anomalous weak values, which are now understood to coherently add up via interference [32] for yielding this unexpected effect. The latter is akin to superoscillations [20, 21] and quantum random walks [22]. Paradoxically, it is interference which gives rise to this single-slit-like behavior, yet a subtle one based on anomalous weak values (indeed, had we blocked the photons emerging from either side, this effect would have disappeared).

Generalizing, for the same setup, but with the initial state

$$|\Psi\rangle = (\alpha|R\rangle - \beta|L\rangle) / \sqrt{\alpha^2 + \beta^2},$$

where $\alpha = \beta + 1 \in \mathbb{R}$, we would observe photons emitted from the anomalously remote location $(\alpha + \beta)d$ (as long as $\lambda \gg (\alpha + \beta)d$). Their time of apparent arrival from R' is therefore similar to that of photons absorbed and re-emitted from locations L and R, giving further credence to the effect. We note that there is a tradeoff between the success probability of the post-selection and the distance of the "phantom position" from the MZI. Superposing the excited atom over N rather than just 2 positions will generally increase the distance at the cost of a lower success probability. Another case of interest is using imaginary α, β, which will create a shift in the momentum distribution rather than the spatial distribution of the emitted photons.

D. Summary

The above analyses demonstrate the significance of anomalous weak values in experiments involving interactions between light and matter. In our case, the anomalous value apparently indicates an atom's negative presence in a certain location, an unusual prediction vindicated with the aid of two thought experiments that reveal two different consequences stemming from it. Importantly, although the light-matter interactions are "strong", the proposed validation techniques, subtly employing quantum uncertainty, are sensitive to the corresponding weak values. Quantum superposition, unique in itself, may therefore be understood as encapsulating some new phenomena emerging upon post-selection. These in turn lead to the novel concept which merits further study, namely counter-particles with negative weak values accompanying particles with positive weak values, together obeying the familiar conservation laws.

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