

Dynamical Couplings and Charge Confinement/Deconfinement from Gravity Coupled to Nonlinear Gauge Fields *

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Received

Abstract. We briefly outline several main results concerning various new physically relevant features found in gravity – both ordinary Einstein or $f(R) = R + R^2$ gravity in the first-order formalism, coupled to a special kind of nonlinear electrodynamics containing a square-root of the standard Maxwell Lagrangian and known to produce charge confinement in flat spacetime.

PACS number: 04.50.-h, 04.70.Bw, 11.25.-w

1 Introduction

G. 't Hooft [1] has shown that in any effective quantum gauge theory, which is able to describe QCD-like charge confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (the latter appearing as a quantum “infrared counterterm”). The simplest way to realize these ideas in flat spacetime is to incorporate into the full gauge field action an additional term being a square-root of the standard Maxwell (or Yang-Mills) gauge field Lagrangian [2–4]:

$$S = \int d^4x L(F^2) \quad , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \quad , \quad (1)$$
$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad .$$

The “square-root” Maxwell term is naturally produced as a result of spontaneous breakdown of scale symmetry in the standard gauge theory [2]. Moreover, the

*Talk at Second Bulgarian National Congress in Physics, Sept. 2013

(dimensionfull) coupling constant f_0 measures the strength of the effective confining potential among quantized fermions produced by (1) [3].

In a series of recent papers [5,6] we have studied in detail physically more interesting models where the above nonlinear “square-root electrodynamics” couples to gravity – either standard Einstein gravity or generalized $f(R)$ -gravity ($f(R)$ being a nonlinear function of the scalar curvature of space-time), as well as coupled to scalar dilaton field. Let us recall that $f(R)$ -gravity models are attracting a lot of interest in modern cosmology as possible candidates to cure problems in the standard cosmological scenarios related to dark matter and dark energy. For a recent review, see e.g. [7] and references therein. The first $R + R^2$ -model (in the second-order formalism) which was also the first inflationary model, was proposed by Starobinsky in [8].

Here we will briefly describe some of our main results [5,6] concerning the new physically relevant features we uncovered in the coupled gravity/“square-root” nonlinear gauge field/dilaton system (defined in Eq.(2) below):

- (i) Appearance of dynamical effective gauge couplings and confinement-deconfinement transition effect as functions of the dilaton vacuum expectation value, in particular due to appearance of “flat” region of the effective dilaton potential.
- (ii) New mechanism for dynamical generation of cosmological constant.
- (iii) Non-standard black hole solutions with constant vacuum radial electric field with Reissner-Nordström-(anti)de-Sitter or Schwarzschild-(anti)de-Sitter type geometry and with non-asymptotically flat “hedgehog”-type spacetime asymptotics. Let us stress that constant vacuum radial electric fields do not exist as solutions of ordinary Maxwell electrodynamics.
- (iv) The above non-standard black holes obey the first law of black hole thermodynamics.
- (v) New “tube-like universe” solutions of Levi-Civita-Bertotti-Robinson type [9];
- (vi) Coupling to *lightlike* branes produces *charge-hiding* and *charge-confining* “thin-shell” wormhole solutions displaying QCD-like charge confinement (see also the previous talk [10] at this conference).

2 $R + R^2$ -Gravity Coupled to Confining Nonlinear Gauge Field

Let us consider coupling of $f(R) = R + \alpha R^2$ gravity (possibly with a bare cosmological constant Λ_0) to a “dilaton” ϕ and the nonlinear gauge field system

containing $\sqrt{-F^2}$ (1) (we are using units with the Newton constant $G_N = 1$):

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi} \left(f(R(g, \Gamma)) - 2\Lambda_0 \right) + L(F^2(g)) + L_D(\phi, g) \right], \quad (2)$$

$$f(R(g, \Gamma)) = R(g, \Gamma) + \alpha R^2(g, \Gamma) \quad , \quad R(g, \Gamma) = R_{\mu\nu}(\Gamma) g^{\mu\nu} \quad , \quad (3)$$

$$L(F^2(g)) = -\frac{1}{4e^2} F^2(g) - \frac{f_0}{2} \sqrt{-F^2(g)} \quad , \quad (4)$$

$$F^2(g) \equiv F_{\kappa\lambda} F_{\mu\nu} g^{\kappa\mu} g^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (5)$$

$$L_D(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \quad . \quad (6)$$

$R_{\mu\nu}(\Gamma)$ is the Ricci curvature in the first order (Palatini) formalism, *i.e.*, the space-time metric $g_{\mu\nu}$ and the affine connection $\Gamma_{\nu\lambda}^\mu$ are *a priori* independent variables. The solution to the corresponding equation of motion w.r.t. $\Gamma_{\nu\lambda}^\mu - \nabla_\lambda(\sqrt{-g} f'_R g^{\mu\nu}) = 0$ – implements transition to the “physical” Einstein-frame metrics $h_{\mu\nu}$ via conformal rescaling of the original metric $g_{\mu\nu}$:

$$g_{\mu\nu} = \frac{1}{f'_R} h_{\mu\nu} \quad , \quad \Gamma_{\nu\lambda}^\mu = \frac{1}{2} h^{\mu\kappa} (\partial_\nu h_{\lambda\kappa} + \partial_\lambda h_{\nu\kappa} - \partial_\kappa h_{\nu\lambda}) \quad . \quad (7)$$

Here $f'_R \equiv \frac{df(R)}{dR} = 1 + 2\alpha R(g, \Gamma)$.

As shown in [6], using (7) the original $R + R^2$ -gravity equations of motion resulting from (2) can be rewritten in the form of *standard* Einstein equations:

$$R_\nu^\mu(h) = 8\pi \left(T_{\text{eff}\nu}^\mu(h) - \frac{1}{2} \delta_\nu^\mu T_{\text{eff}\lambda}^\lambda(h) \right) \quad (8)$$

with effective energy-momentum tensor of the following form:

$$T_{\text{eff}\mu\nu}(h) = h_{\mu\nu} L_{\text{eff}}(h) - 2 \frac{\partial L_{\text{eff}}}{\partial h^{\mu\nu}} \quad . \quad (9)$$

The pertinent effective “Einstein-frame” matter Lagrangian reads:

$$\begin{aligned} L_{\text{eff}}(h) = & -\frac{1}{4e_{\text{eff}}^2(\phi)} F^2(h) - \frac{1}{2} f_{\text{eff}}(\phi) \sqrt{-F^2(h)} \\ & + \frac{X(\phi, h) (1 + 16\pi\alpha X(\phi, h)) - V(\phi) - \Lambda_0/8\pi}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} \end{aligned} \quad (10)$$

with the following dynamical ϕ -dependent couplings:

$$\frac{1}{e_{\text{eff}}^2(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} \quad , \quad (11)$$

$$f_{\text{eff}}(\phi) = f_0 \frac{1 + 32\pi\alpha X(\phi, h)}{1 + 8\alpha (8\pi V(\phi) + \Lambda_0)} \quad . \quad (12)$$

The dilaton kinetic term $X(\phi, h) \equiv -\frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ will be ignored in the sequel.

3 Dynamical Couplings and Confinement/Deconfinement in $R+R^2$ Gravity

In what follows we consider constant “dilaton” ϕ extremizing the effective Lagrangian (10). Here we observe an interesting feature of (10) – the dynamical couplings and effective potential are extremized *simultaneously*, which is an explicit realization of the so called “least coupling principle” of Damour-Polyakov [11]:

$$\frac{\partial f_{\text{eff}}}{\partial \phi} = -64\pi\alpha f_0 \frac{\partial V_{\text{eff}}}{\partial \phi} , \quad \frac{\partial}{\partial \phi} \frac{1}{e_{\text{eff}}^2} = -(32\pi\alpha f_0)^2 \frac{\partial V_{\text{eff}}}{\partial \phi} \rightarrow \frac{\partial L_{\text{eff}}}{\partial \phi} \sim \frac{\partial V_{\text{eff}}}{\partial \phi} , \quad (13)$$

where:

$$V_{\text{eff}}(\phi) = \frac{V(\phi) + \frac{\Lambda_0}{8\pi}}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)} . \quad (14)$$

Thus, at a constant extremum of L_{eff} (10) ϕ must satisfy:

$$\frac{\partial V_{\text{eff}}}{\partial \phi} = \frac{V'(\phi)}{[1 + 8\alpha(8\pi V(\phi) + \Lambda_0)]^2} = 0 . \quad (15)$$

There are two generic cases:

(a) *Confining phase*: Eq.(15) is satisfied for some finite-value ϕ_0 extremizing the original “bare” dilaton potential $V(\phi)$: $V'(\phi_0) = 0$.

(b) *Deconfinement phase*: For polynomial or exponentially growing “bare” potential $V(\phi)$, so that $V(\phi) \rightarrow \infty$ when $\phi \rightarrow \infty$, we have:

$$\frac{\partial V_{\text{eff}}}{\partial \phi} \rightarrow 0 \quad , \quad V_{\text{eff}}(\phi) \rightarrow \frac{1}{64\pi\alpha} = \text{const} \quad \text{when } \phi \rightarrow \infty , \quad (16)$$

i.e., for sufficiently large values of ϕ we find a “flat region” in V_{eff} . This “flat region” triggers a *transition from confining to deconfinement dynamics*.

Namely, in the “flat-region” phase (b) we have $f_{\text{eff}} \rightarrow 0$, $e_{\text{eff}}^2 \rightarrow e^2$, and the effective gauge field Lagrangian in (10) reduces to the ordinary *non-confining* one (the “square-root” Maxwell term $\sqrt{-F^2}$ vanishes):

$$L_{\text{eff}}^{(0)} = -\frac{1}{4e^2} F^2(h) - \frac{1}{64\pi\alpha} \quad (17)$$

with an *induced* cosmological constant $\Lambda_{\text{eff}} = 1/8\alpha$, which is *completely independent* of the bare cosmological constant Λ_0 .

4 Non-Standard Black Holes and “Tube-Like” Universes

In the *confining phase* (phase (a) above: ϕ_0 – generic minimum of the effective dilaton potential; $e_{\text{eff}}(\phi)$, $f_{\text{eff}}(\phi)$ as in (11)–(12)) we obtain several physically

interesting solutions of the Einstein-frame gravity Eqs.(8) due to the special form of the effective Einstein-frame matter Lagrangian (10).

First, we find *non-standard* Reissner-Nordström-(anti-)de-Sitter-type black holes with metric:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (18)$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f_{\text{eff}}(\phi_0)e_{\text{eff}}(\phi_0) - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}(\phi_0)}{3}r^2, \quad (19)$$

where $\Lambda_{\text{eff}}(\phi_0)$ is a *dynamically generated* cosmological constant:

$$\Lambda_{\text{eff}}(\phi_0) = \frac{\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2}{1 + 8\alpha(\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2)}. \quad (20)$$

The black hole's static radial electric field contains apart from the Coulomb term an *additional constant "vacuum" piece*:

$$|F_{0r}| = |\vec{E}_{\text{vac}}| - \frac{Q}{\sqrt{4\pi}r^2} \left(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha(8\pi V(\phi_0) + \Lambda_0)} \right)^{-\frac{1}{2}} \quad (21)$$

$$|\vec{E}_{\text{vac}}| \equiv \left(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha(8\pi V(\phi_0) + \Lambda_0)} \right)^{-1} \frac{f_0/\sqrt{2}}{1 + 8\alpha(8\pi V(\phi_0) + \Lambda_0)}. \quad (22)$$

Let us emphasize again that constant vacuum radial electric fields do not exist as solutions of ordinary Maxwell electrodynamics.

In the special case $\Lambda_{\text{eff}}(\phi_0) = 0$ we obtain a non-standard Reissner-Nordström-type black hole with a "hedgehog" *non-flat-spacetime* asymptotics (cf. Refs. [12]: $A(r) \rightarrow 1 - \sqrt{8\pi}|Q|f_{\text{eff}}(\phi_0)e_{\text{eff}}(\phi_0) \neq 1$ for $r \rightarrow \infty$).

Apart from non-standard black hole solutions we also obtain "tube-like" space-time solutions of the Einstein-frame gravity Eqs.(8), which are of Levi-Civita-Bertotti-Robinson type (cf. Refs. [9]) with geometries $AdS_2 \times S^2$, $Rind_2 \times S^2$ and $dS_2 \times S^2$, where AdS_2 , $Rind_2$ and dS_2 are two-dimensional anti-de Sitter, Rindler and de Sitter space, respectively. The corresponding metric is of the form:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad -\infty < \eta < \infty, \quad (23)$$

carrying constant vacuum "radial" electric field $|F_{0\eta}| = |\vec{E}_{\text{vac}}|$. The radius of the spherical factor S^2 is given by (using short-hand $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$):

$$\frac{1}{r_0^2} = \frac{4\pi}{1 + 8\alpha\Lambda(\phi_0)} \left[\left(1 + 8\alpha(\Lambda(\phi_0) + 2\pi f_0^2) \right) \vec{E}_{\text{vac}}^2 + \frac{1}{4\pi}\Lambda(\phi_0) \right], \quad (24)$$

and the metric coefficient in (23) reads: $A(\eta) = 4\pi K(\vec{E}_{\text{vac}})\eta^2$, $K(\vec{E}_{\text{vac}}) > 0$ for AdS_2 ; $A(\eta) = \pm\eta$ for $\eta \in (0, +\infty)$ or $\eta \in (-\infty, 0)$ for $Rind_2$; $A(\eta) =$

$1 - 4\pi|K(\vec{E}_{\text{vac}})|\eta^2$, $K(\vec{E}_{\text{vac}}) < 0$ for dS_2 , using notation:

$$K(\vec{E}) \equiv \left(1 + 8\alpha(\Lambda(\phi_0) + 2\pi f_0^2)\right)\vec{E}^2 - \sqrt{2}f_0|\vec{E}| - \frac{1}{4\pi}\Lambda(\phi_0).$$

To conclude, let us particularly stress that all physically relevant features described above are exclusively due to the *combined* effect of both square-root nonlinearity $\sqrt{-F^2}$ in the gauge field Lagrangian as well as the R^2 term in the gravity action.

Acknowledgments

We gratefully acknowledge support of our collaboration through the academic exchange agreement between the Ben-Gurion University and the Bulgarian Academy of Sciences. S.P. has received partial support from COST action MP-1210.

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